# Shear and Vorticity in a Combined Einstein-Cartan-Brans-Dicke Inflationary Lambda-Universe

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## Abstract

A combined BCDE (Brans-Dicke and Einstein-Cartan) theory with lambda-term is developed through Raychaudhuri's equation, for inflationary scenario. It involves a variable cosmological constant, which decreases with time, jointly with energy density, cosmic pressure, shear, vorticity, and Hubble's parameter, while the scale factor, total spin and scalar field increase exponentially. The post-inflationary fluid resembles a perfect one, though total spin grows, but the angular speed does not (Berman, 2007d).

Keywords: Cosmology; Einstein; Brans-Dicke; Cosmological term; Shear; Spin; Vorticity; Inflation; Einstein-Cartan; Torsion.

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Berman(2007b), examined the time behavior of shear and vorticity in a lambda-Universe, for inflationary models, in a Brans-Dicke framework. The resulting scenario is that exponential inflation smooths the fluid, in order to become a nearly perfect one after the inflationary period. We now examine the inclusion of spin, by means of Einstein-Cartan's theory, when a scalar field of Brans-Dicke origin, is included, along with a Cosmological lambda-term.

Einstein-Cartan's gravitational theory, though not bringing vacuum solutions different than those in General Relativity theory, has an important rôle, by tying macrophysics, through gravitational and electromagnetic phenomena (i.e., involving constants $G$ and $c$), with microphysics, though Planck's constant, involving spin originated by torsion. Intrinsic angular momentum was introduced by Cartan as a Classical quantity (Cartan, 1923) before it was introduced as a Quantum Theory element, around 1925. Of course, spin is important in the Quantum Theory of particles. However, spin has taken part of Classical Field Theory for a long time, and Cosmological models were treated as early as 1973 (Trautman, 1973). Einstein-Cartan Theory is the simplest Poincaré gauge theory of gravity, in the frame of which, the gravitational field is described by means of curvature and torsion, the sources being energy-momentum and spin tensors. It is important to stress that torsion can be originated by spin but not necessarily vice-versa.

Though it was in the past, supposed that, due to spin, Robertson-Walker's metric might not be representative of Physical reality in a torsioned spacetime, recent papers recalled the approach shown by us in several papers (Berman, 1990; 1991), on how anisotropic Bianchi-I models in Einstein-Cartan's theory could be reduced to Robertson-Walker's prototype, by defining overall, deceleration parameters, and scale-factors; we did the same thing, with other papers dealing with anisotropic models in GRT and BD theories [ for GRT see (Berman, 1988; Berman and Som, 1989 b); for BD theory see (Berman and Som,1989) ]. On

the other hand, Berman and Som (2007) have shown that, slight deviations from Robertson-Walker's metric, changing it to a Bianchi-I metric, are enough to produce the anisotropic phenomena, like entropy production, or other ones; this is a clue to the possibility of considering overall scale-factors and deceleration parameters, etc, in the Raychaudhuri's equation for Einstein-Cartan's Cosmology, without worrying with any anisotropy, which becomes implicit in the equations of Raychaudhuri's book (Raychaudhuri, 1979). The essential modification of General Relativistic Bianchi-I cosmology, when we carry towards Einstein-Cartan's, resides, when field equations are explicited, in that the normal energy momentum tensor components $T_1^1$, $T_2^2$ and $T_3^3$ are subtracted by a term $S^2$, while $T_0^0$ is added by $S^2$. Of course, there appear also non-diagonal $S-$ dependent terms: for instance, $T_3^2$ and $T_2^3$ depend linearly with $S^{32}$. In our treatment of the Einstein-Cartan-Brans-Dicke theory, the field equations are obviously satisfied, but we have short-cutted the derivations, like we have done in the previous paper (Berman, 2007b), which also conforms with the field equations of that case (Brans-Dicke theory with lambda). The off-diagonal energy momentum components are null, for a Robertson-Walker's framework.

It is generally accepted that scalar tensor cosmologies play a central rôle in the present view of the very early Universe (Berman, 2007). The cosmological "constant", which represents quintessence, is a time varying entity, whose origin remounts to Quantum theory(Berman, 2007a). The first, and most important scalar tensor theory was devised by Brans and Dicke(1961), which is given in the "Jordan's frame". Afterwards, Dicke(1962) presented a new version of the theory, in the "Einstein's frame", where the field equations resembled Einstein's equations, but time, length, and inverse mass, were scaled by a factor $\phi^{-\frac{1}{2}}$ where $\phi$ stands for the scalar field. Then, the energy momentum tensor $T_{ij}$ is augmented by a new term $\Lambda_{ij}$, so that:

$$G_{ij} = -8\pi G \left(T_{ij} + \Lambda_{ij}\right) \qquad , \qquad (1)$$

where $G_{ij}$ stands for Einstein's tensor. The new energy tensor quantity, is given by:

$$\Lambda_{ij} = \frac{2\omega+3}{16\pi G\phi^2} \left[\phi_i\phi_j - \frac{1}{2}G_{ij}\phi_k\phi^k\right] \qquad . \qquad (2)$$

In the above, $\omega$ is the coupling constant. The other equation is:

$$\Box \log \phi = \frac{8\pi G}{2\omega+3} T \quad , \tag{3}$$

where $\Box$ is the generalized d'Alembertian, and $T = T_i^i$ . It is useful to remember that the energy tensor masses are also scaled by $\phi^{-\frac{1}{2}}$ .

For the Robertson-Walker's flat metric,

$$ds^2 = dt^2 - \frac{R^2(t)}{\left[1+\left(\frac{kr^2}{4}\right)\right]^2} d\sigma^2 \quad , \tag{4}$$

where $k = 0$ and $d\sigma^2 = dx^2 + dy^2 + dz^2$ .

The field equations now read, in the alternative Brans-Dicke reformulation(Raychaudhuri, 1979):

$$\frac{8\pi G}{3}\left(\rho + \frac{\Lambda}{\kappa} + \rho_\lambda\right) = H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 \quad . \tag{5}$$

$$-8\pi G \left(p - \frac{\Lambda}{\kappa} + \rho_\lambda\right) = H^2 + \frac{2\ddot{R}}{R} \quad . \tag{6}$$

In the above, we have:

$$\rho_\lambda = \frac{2\omega+3}{32\pi G}\left(\frac{\dot{\phi}}{\phi}\right)^2 = \rho_{\lambda 0}\left(\frac{\dot{\phi}}{\phi}\right)^2 \quad . \tag{7}$$

From the above equations (5), (6) and (7) we obtain:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + 3p + 4\rho_\lambda - \frac{\Lambda}{4\pi G}\right) \quad . \tag{8}$$

Relation (8) represents Raychaudhuri's equation for a perfect fluid. By the usual procedure, we would find the Raychaudhuri's equation in the general case, involving shear $(\sigma_{ij})$ and vorticity $(\varpi_{ij})$; the acceleration of the fluid is null for the present case, and then we find:

$$3\dot{H} + 3H^2 = 2\left(\varpi^2 - \sigma^2\right) - 4\pi G\left(\rho + 3p + 4\rho_\lambda\right) + \Lambda \quad , \tag{9}$$

where $\Lambda$ stands for a cosmological "constant". As we are mimicking Einstein's field equations, $\Lambda$ in (9) stands like it were a constant (see however, Berman, 2007, 2007a, 2006b, 2006c). Notice that, when we impose that the fluid is not accelerating, this means that the quadri-velocity is tangent to the geodesics, i.e., the only interaction is gravitational.

When Raychaudhuri's equation is calculated for non-accelerated fluid, taken care of Einstein-Cartan's theory, combined with Brans-Dicke theory, the following equation was found by us, based on the original calculation for Einstein-Cartan's theory by Raychaudhuri (1979):

$$3\dot{H} + 3H^2 = 2\varpi^2 - 2\sigma^2 - 4\pi G\left(\rho + 3p + 4\rho_\lambda\right) + \Lambda + 128\pi^2 S^2 \qquad , \quad (10)$$

where $S$ stands for the spin density contents of the fluid, where we have omitted a term like

$$\varpi S = \varpi_{ik}S^{ik} + \varpi^{ik}S_{ik} \qquad , \qquad (10a)$$

which is to be included in the pressure and energy density terms, by a re-scaling.

It is important to stress, that relation (10) is the same general relativistic equation, with the additional spin term, which transforms it into Einstein-Cartan's equation. When we work a combined Einstein-Cartan's and Brans-Dicke theory (BCDE theory), we would need to calculate the new field equations for the combined theory.

By employing the total action (Sabbata and Gasperini, 1985),

$$L = \int d^4x\sqrt{-g}\left[\pounds_m\left(\psi, \nabla\psi, g\right) - \tfrac{1}{2\chi}R\left(g, \partial g, Q\right)\right] \qquad , \qquad (10b)$$

where the matter Lagrangian contains torsion because the connection is not symmetric, and $\chi$ is the coupling constant, both for curvature and torsion, and when we perform independent variations with respect to $\psi$ , $g_{\mu\nu}$ and $Q_{\mu\nu}^{\alpha}$ ; the last the one is the torsion tensor,

$$Q_{\alpha\beta}^{\mu} = \tfrac{1}{2}\left(\Gamma_{\alpha\beta}^{\mu} - \Gamma_{\beta\alpha}^{\mu}\right) \qquad . \qquad (10c)$$

We find, the Einstein tensor,

$$G^{\mu\nu} - \hat{\nabla}_\alpha\left(T^{\mu\nu\alpha} - T^{\nu\alpha\mu} + T^{\alpha\mu\nu}\right) = \chi T^{\mu\nu} \qquad , \qquad (10d)$$

where,

$$T^{\mu\nu\alpha} = \chi S^{\mu\nu\alpha} \qquad . \qquad (10e)$$

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We have defined,

$$\hat{\nabla}_\alpha \equiv \nabla_\alpha + 2Q_\alpha = \nabla_\alpha + 2Q_{\alpha\nu}^\nu \qquad , \qquad \text{(10f)}$$

while the modified torsion tensor,

$$T_{\mu\nu}^\alpha = Q_{\mu\nu}^\alpha + \delta_\mu^\alpha Q_\nu - \delta_\nu^\alpha Q_\mu \qquad . \qquad \text{(10g)}$$

The resulting equations for a perfect fluid, can be found in Raychaudhuri (1979):

$$-8\pi p = [\text{Brans-Dicke alternative Riemann tensor } G_i^i \,] + 256\pi^2 S^2 \quad , \quad \text{(10h)}$$

$$8\pi\rho = [\text{Brans-Dicke alternative Riemann tensor } G_0^0 \,] + 256\pi^2 S^2 \qquad , \quad \text{(10i)}$$

It is important to acknowledge, that the above field equations should be applied into the pseudo-General Relativistic equations, i.e., the Brans-Dicke alternative (unconventional) framework. A plausibility reasoning that substitutes an otherwise lengthy calculation, is the following: the term with spin, as well as it is added to the other general relativistic terms in equation (10), should be added equally to equation (9), because this is the Brans-Dicke equation in a general relativistic format. This equation is written in the unconventional format (Dicke, 1962), i.e., the alternative system of equations. We could not write so simply equation (10) if the terms in it were those of conventional Brans-Dicke theory.

Consider now exponential inflation, like we find in Einstein's theory:

$$R = R_0 e^{Ht} \qquad , \qquad \text{(11)}$$

and, as usual in General Relativity inflationary models,

$$\Lambda = 3H^2 \qquad . \qquad \text{(12)}$$

For the time being, $H$ is just a constant, defined by $H = \frac{\dot{R}}{R}$ . We shall see, when we go back to conventional Brans-Dicke theory, that $H$ is not the Hubble's constant.

From (11), we find $H = H_0 = \text{constant}$.

A solution of Raychaudhuri's equation (10), would be the following:

$$\sigma = \sigma_0 e^{-\frac{\beta}{2}t} \qquad ;$$

$$\varpi = \varpi_0 e^{-\frac{\beta}{2}t} \quad ;$$

$$\rho = \rho_0 e^{-\beta t} \quad ;$$

$$p = p_0 e^{-\beta t} \quad ; \tag{13}$$

$$\phi = \phi_0 e^{-\frac{\beta}{2}\sqrt{A}} e^{-\frac{\beta}{2}t} \quad .$$

$$\Lambda = \Lambda_0 = \text{constant.}$$

$$S_U = SR^3 = s_0 R_0^3 e^{Ht} \quad .$$

In the above, $\sigma_0$ , $\phi_0$ , $p_0$ , $\rho_0$ , $\beta$ , $s_0$ and $R_0$ , are constants, and, $S_U$ stands for the total spin of the Universe, whose spin density equals,

$$S = s_0 e^{-\frac{\beta}{2}t} = s_0 e^{-2Ht} \quad , \tag{14}$$

while,

$$\beta = 4H \quad . \tag{15}$$

The ultimate justification for this solution is that one finds a good solution in the conventional units theory, and that the Universe must expand.

When we return to conventional units, we retrieve the following corresponding solution:

$$\bar{R} = R_0 \phi^{\frac{1}{2}} e^{Ht} \quad ;$$

$$\bar{\rho} = \rho_0 \phi^{-2} e^{-\beta t} \quad ;$$

$$\bar{p} = p_0 \phi^{-2} e^{-\beta t} = \left[\frac{p_0}{\rho_0}\right] \bar{\rho} \quad ;$$

$$\tag{16}$$

$$\bar{\sigma} = \sigma \phi^{-\frac{1}{2}} \quad ;$$

$$\bar{\varpi} = \varpi \phi^{-\frac{1}{2}} \quad ;$$

$$\bar{\Lambda} = \Lambda_0 \phi^{-1} \quad ;$$

$$\bar{\phi} = \phi = \phi_0 e^{-\frac{\beta}{2}\sqrt{A}} \ e^{-\frac{\beta}{2}t} \qquad .$$

We also have,

$$\bar{S}_U = S_U = s_0 R_0^3 e^{Ht} \qquad , \qquad \text{in} \qquad c = 1 \quad \text{units} \qquad . \tag{17}$$

As we promised to the reader, $H$ is not the Hubble's constant. Instead, we find:

$$\bar{\Lambda} = \Lambda_0 \ \phi_0^{-1} \ e^{\frac{\beta}{2}\sqrt{A}} e^{-\frac{\beta}{2}t} \qquad ; \tag{18}$$

$$\bar{\rho} = \rho_0 \ \phi_0^{-2} \ e^{\beta\left[\sqrt{A} \ e^{-\frac{\beta}{2}t} - t\right]} \qquad ; \tag{19}$$

$$\bar{p} = p_0 \ \phi_0^{-2} \ e^{\beta\left[\sqrt{A} \ e^{-\frac{\beta}{2}t} - t\right]} \qquad ; \tag{20}$$

$$\bar{R} = R_0 \ \phi_0^{-\frac{1}{2}} \ e^{\left[H t - \frac{1}{4}\beta \sqrt{A} \ e^{-\frac{\beta}{2}t}\right]} \qquad ; \tag{21}$$

$$\bar{\sigma} = \sigma_0 \ \phi_0^{-\frac{1}{2}} \ e^{-\frac{1}{2}\beta\left[t - \frac{1}{2} \sqrt{A} \ e^{-\frac{\beta}{2}t}\right]} \qquad , \tag{22}$$

$$\bar{\varpi} = \varpi_0 \ \phi_0^{-\frac{1}{2}} \ e^{-\frac{1}{2}\beta\left[t - \frac{1}{2} \sqrt{A} \ e^{-\frac{\beta}{2}t}\right]} \qquad , \tag{23}$$

and,

$$\bar{H} = H \ \phi_0^{-\frac{1}{2}} \ e^{\frac{1}{4}\beta\sqrt{A}} \ e^{-\frac{\beta}{2}t} > 0 \qquad . \tag{23 a}$$

The fluid obeys a perfect gas equation of state. It represents a radiation phase, if we impose,

$$p_0 = \tfrac{1}{3}\rho_0 \qquad . \tag{24}$$

Returning to Raychaudhuri's equation, we have the following condition to be obeyed by the constants:

$$\sigma_0^2 - \varpi_0^2 = -2\pi G \left[\rho_0 + 3p_0 + 4\rho_{\lambda 0}\right] + 64\pi^2 s_0^2 \qquad . \tag{25}$$

We now investigate the limit when $t \longrightarrow \infty$ of the above formulae, having in mind that, by checking that limit, we will know which ones increase or decrease with time; of

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course, we can not stand with an inflationary period unless it takes only an extremely small period of time. Remember that $\beta = 4H > 0$ .

We find:

$$\lim_{t \longrightarrow \infty} \bar{H} = H\phi_0^{-1/2} \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{R} = \infty \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{\sigma} = \lim_{t \longrightarrow \infty} \bar{\varpi} = 0 \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{\rho} = \lim_{t \longrightarrow \infty} \bar{p} = 0 \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{\Lambda} = \Lambda_0 \phi_0^{-1} \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{\phi} = \phi_0 \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{S}_U = \infty \quad .$$

By comparing the above limits, with the limit $t \to 0$ , as we can check, the scale factor, total spin, and the scalar field, are time-increasing, while all other elements of the model, namely, vorticity, shear, Hubble's parameter, energy density, cosmic pressure, and cosmological term, as described by the above relations, decay with time. This being the case, shear and vorticity are decaying, so that, after inflation, we retrieve a nearly perfect fluid: inflation has the peculiarity of removing shear, and vorticity, but not spin, from the model. It has to be remarked, that pressure and energy density obey a perfect gas equation of state. The graceful exit from the inflationary period towards the early Universe radiation phase, is attained with condition (24). We have found a solution that is entirely compatible with the Brans-Dicke counterpart (Berman, 2007c). The total spin of the Universe grows, but the angular velocity does not (Berman, 2007d).

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